

Analytic Description of Hypermixing and Test of an Improved Nozzle

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A novel method for increasing the rate of jet mixing by the production of streamwise vortices has been developed. An analytic description of this hypermixing effect is obtained with an eddy viscosity whose length and velocity scales are proportional to the vortex size and rotational speed. The predictions of this analysis are compared with experimental results for the growth of hypermixing jets, and the generalization to other shear flows is discussed. Tests of an improved hypermixing nozzle show that the length of the thrust augmenting ejector developed at ARL can be almost halved, with no loss in augmentation.

Nomenclature

A	= area
d	= nozzle exit width
f	= similarity profile function
F	= similarity stream function
I	= momentum integral
L	= length scale
ℓ	= hypermixing length
v	= hypermixing velocity
r	= radial coordinate
T	= time of flight variable
U, V	= mean velocity components
u, v	= fluctuating velocity components
x, y	= cartesian coordinates
α	= constant of proportionality
ϵ	= turbulent viscosity
η	= similarity parameter
ξ	= coordinate transformation
ϕ	= velocity transformation
ψ	= stream function
Ω	= hypermixing vorticity

Subscripts

0	= nozzle exit value
I	= initial value
c	= characteristic value

Superscripts

$\bar{}$	= mean value
\prime	= differentiation

Introduction

IT is possible for aircraft to develop the additional thrust required for vertical or very short takeoff and landing from the cruise engine by diverting the exhaust jet through a thrust augmenting ejector. An ejector is a mechanically simple device in which turbulent entrainment by a jet of primary fluid is used to pump a secondary flow through a duct. In aircraft applications an ejector functions like a fan or low-pressure-ratio compressor to increase the static thrust of the basic turbofan engine, see Fig. 1.

The shroud of the simple ejector in Fig. 1 consists of a contoured inlet section, a mixing duct, and a diffuser. Entrainment by the engine exhaust jet induces a secondary flow

through the shroud. The primary jet is decelerated as it mixes with the secondary flow, causing the average static (and total) pressure in the duct to rise. Since the primary flow is only a fraction of the secondary flow, the mixing section may be considered to act as an elongated actuator disk, or fan. Because of the increased static pressure in the diffuser section, the inlet-diffuser venturi develops a net lip thrust. It is this force that augments the thrust of the primary jet.

When integrated with the wing into a lift propulsion system, the ejector acts like a jet flap to increase the circulation lift during transition. Since ejectors can be used to augment and deflect the engine thrust during takeoff and landing, their use permits the installation of a smaller engine for more efficient cruise. The lowered exhaust velocities and acoustic shielding provided by the walls of the duct offer important advantages in the area of noise reduction, as well.

The application of ejector thrust augmentation to V/STOL aircraft has, therefore, been an active research goal for several decades. However, complete mixing of the primary jet and secondary entrainment flow is required to achieve significant levels of augmentation. In the past this has meant that an ejector must include a long mixing duct. Ejectors shortened to meet aircraft size and weight limitations could not generate sufficient thrust to make the concept practical.

The performance of short ejectors can be increased by accelerating the rate of entrainment by the primary jet (hypermixing). The hypermixing nozzles developed at ARL produce a system of streamwise vortices in the primary flow. These vortices serve to entrain additional fluid, so that a more fully mixed flow is obtained within a shorter distance. In the present article an eddy viscosity for hypermixing jets is presented, and tests of an improved nozzle, which has made large reductions in ejector length possible, are reported.

Hypermixing Effect

Although considerable effort has gone into the study of the turbulent mixing process, it is still not clear how entrainment

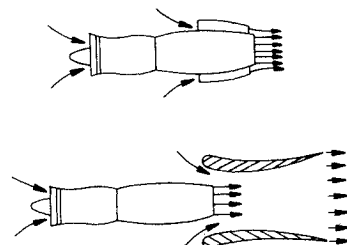


Fig. 1 Aft-fan turbofan engine and turbojet engine with simple ejector.

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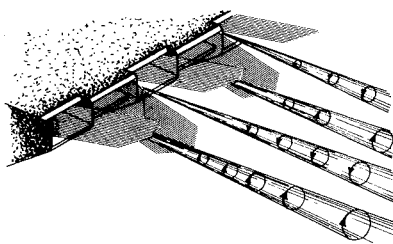


Fig. 2 Alternating exit of hypermixing nozzle.

occurs. It is reasonable, however, to suppose that the rate at which a turbulent jet spreads depends upon the scale and intensity of the turbulence. Producing streamwise vorticity amounts to redistributing the energy of the flow to control the turbulence. In the hypermixing nozzles developed for the present work, this is accomplished by segmenting the exit plane of a slot nozzle, as shown in Fig. 2. The flow on the shorter side of each element expands to atmospheric pressure more rapidly than on the longer side. The resultant local pressure difference deflects the segment of the jet to the short side, as shown. By alternating the surfaces which are cut back, the desired vorticity is produced.

The ability of turbulence to transport momentum across a mean velocity gradient is ascribed to the turbulent convection of fluid elements from one part of the mean velocity profile to another. The hypermixing vortices can be considered to enhance this process. It should be noted that the hypermixing mechanism is distinct from the swirl sometimes used to accelerate the spread of axisymmetric jets. The swirling process merely increases the shear at the jet boundary, without producing any transport across the velocity profile.

The hypermixing vortices will be treated as a significant turbulence component which generates an additional Reynolds stress. The equation of motion for the two-dimensional, hypermixing jet thus takes the form

$$U(\partial U/\partial x) + V(\partial U/\partial y) = (\partial/\partial y)(\bar{u}v + \bar{u}v)$$

The vortex induced stress $\bar{u}v$ represents momentum of the fluid u , convected at the rotational speed v , characteristic of the hypermixing vortex. It is similar to the ordinary turbulent shear stress $\bar{u}v$, and in the present case results in a streamwise retarding force on a unit volume of fluid.

Since the most successful treatments of jet mixing have been in the form of an experimentally determined effective viscosity, this same pragmatic approach will be adopted here. The highly three-dimensional character of this flow is thus ignored in the interest of mathematical simplicity. However, the essential feature of hypermixing flows is preserved by relating the mixing length and velocity of the eddy viscosity to the vortex size and rotational speed.

If it is assumed that the velocity fluctuations u are due to the transport of fluid elements across the jet, the magnitude of these fluctuations must be on the order of the velocity difference across the jet; that is, $u = \ell \partial U/\partial y$, where ℓ is the distance over which an element has been transported. The hypermixing stress may, therefore, be written

$$(\partial/\partial y)(\bar{u}v) = \bar{v} \partial^2 U/\partial y^2$$

This is formally identical to Taylor's¹ and Prandtl's² mixing length expression for the turbulent shear stress. The present analysis will differ in the relations used for the mixing length and velocity.

The diffusion of the hypermixing vortices and the spreading of the jet are coupled. The vortices increase the rate of turbulent mixing, but they are, in turn, diffused by the turbulence and strained by the mean jet flow. The equation describing the change in the streamwise vorticity Ω is

$$D\Omega/Dt = (\partial U/\partial x)\Omega + \epsilon \nabla^2 \Omega$$

This is a statement that the rate of change of vorticity ($D\Omega/Dt$) is due to straining by the mean flow ($\partial U/\partial x$) and diffusion by the turbulence ($\epsilon \nabla^2 \Omega$).

The deceleration of the jet produces a mean compressive rate of strain on the vortices. The turbulent diffusion of the vortices can be expressed as a function of displacement downstream if it is assumed that the lines of circulation are simply convected by the mean flow, $D/Dt = U_c \partial/\partial x$. The vorticity equation thus becomes

$$U_c \partial \Omega/\partial x = (\partial U_c/\partial x)\Omega + \epsilon \nabla^2 \Omega$$

which will be solved for the streamwise vorticity as a function of the characteristic jet velocity U_c .

Consider, first, the case of pure straining, $U d\Omega = \Omega dU$. This is a simple linear differential equation with solution

$$\Omega(x) = \Omega_1 U_c(x)/U_1$$

in which Ω_1 and U_1 are the initial values of these parameters. It will be assumed that the solution of the complete equation has the form

$$\Omega(x) = \Omega_1(x) U_c(x)/U_1$$

that is, the change in streamwise vorticity is the product of turbulent diffusion and the compressive rate of strain.

Substituting in the complete vorticity equation yields an equation for $\Omega_1(x)$

$$U_c \partial \Omega_1/\partial x = \epsilon \nabla^2 \Omega_1$$

This equation can be solved by a change of variables. Let

$$\Omega_1 = (\partial \phi/\partial r) r^{-1} + \partial^2 \phi/\partial r^2$$

integrating once yields an equation similar to the heat conduction equation, which, therefore, has a similar solution

$$\phi = AT^{-1} \exp(-r^2/4\epsilon T)$$

A is a constant with units of an area and T is the effective age of the vortex,

$$T = \int_0^x U_c^{-1} dx$$

The velocity distribution in the simultaneously strained and diffusing vortex can be obtained by integrating the expression for the vorticity product. The details of this procedure can be found in Ref. 3. The distribution obtained is

$$v(r, x) = (Ar/2 \epsilon T^2) (U_c/U_1) \exp(-r^2/4 \epsilon T)$$

The size of the vortex will be defined as the radius at which the angular velocity is maximum

$$r_c = (2 \epsilon T)^{1/2}$$

This maximum velocity is

$$v_c = (A/e^{1/2}) (U_c/U_1) (2 \epsilon T^3)^{-1/2}$$

and will be taken as characteristic of the speed at which the hypermixing vortex rotates. If the mixing length and velocity scales ℓ and v are assumed to be proportional to the vortex size and rotational speed, the vortex induced stress can then be written

$$(\partial/\partial y) \bar{u}v = (A_c/T) (U_c/U_1) \partial^2 U/\partial y^2$$

A_c is a constant that depends on the initial size and strength of the vortex, and must be evaluated from experiment. Note that this stress is independent of the coefficient of diffusivity ϵ . The coupling between the jet and vortex motions is thus included in the momentum equation through the additional eddy viscosity. It is then, of particular interest to determine whether a simple self-preserving solution of this equation is possible, and to compare the predicted behavior to experimental results.

Solution of Equations of Motion

The continuity equation will be integrated by the use of stream function defined in the usual way; i.e., such that $U = \partial\psi/\partial y$ and $V = -\partial\psi/\partial x$. Substituting into the momentum equation then yields

$$\frac{\partial\psi}{\partial y} \frac{\partial^2\psi}{\partial x\partial y} - \frac{\partial\psi}{\partial x} \frac{\partial^2\psi}{\partial x\partial y} = \epsilon(x) \frac{\partial^3\psi}{\partial y^3}$$

in which $\epsilon(x)$ is the hypermixing eddy viscosity and includes both the usual turbulent stress and the effect of the stream-wise vortices. A solution to this equation can be found through the use of a coordinate transformation, $\xi(x)$, defined such that

$$\partial/\partial x = (\partial\xi/\partial x) \partial/\partial\xi$$

Self-preservation means that the shape of the velocity profile is independent of x when expressed in terms of local length and velocity scales L_c and U_c that is, $U/U_c = f(y/L_c)$. It is plausible to assume the growth of L_c as ξ^m and the decay of U_c as ξ^{-n} or, if distances are nondimensionalized with the width of the nozzle exit

$$L_c = L_I (\xi/d)^m \text{ and } U_c = U_I (\xi/d)^{-n}$$

Then, since

$$\partial\psi/\partial y \equiv U_I (\xi/d)^{-n} f(y/L_c)$$

the stream function must have the form

$$\psi = U_I L_I (\xi/d)^{m-n} F(\eta)$$

which may be verified by differentiation (η is the similarity parameter, $\eta = y/L_c$, and $f = F'$). The eddy viscosity for the ordinary turbulent stress is defined in the usual way, $\epsilon = \alpha U_c L_c$, and so the hypermixing eddy viscosity becomes

$$\epsilon(x) = \alpha L_I U_I (\xi/d)^{m-n} + A_c T^{-1} (\xi/d)^{-n}$$

Substituting these expressions into the equation of motion yields the following differential equation for $F(\eta)$ and $\xi(x)$

$$(L_I/d F''') (\xi/d)^{m-1} \{ -n F' F' - (m-n) F F'' \} \\ = [\alpha + A_c T^{-1} (\xi/d)^{-m}] / (d\xi/dx)$$

If the shape of the velocity profile is to be self-preserving, the left-hand side must be a function of η only. This requires that $m=1$. The second constraint needed to evaluate n is that the momentum integral must be constant. If the flow becomes self-preserving, this requirement is

$$L_c U_c^2 \int_{-\infty}^{\infty} (F')^2 d\eta = U_0^2 d$$

The product

$$L_c U_c^2 = L_I U_I^2 (\xi/d)^{m-2n}$$

is independent of x when $n=1/2$. U_0 is the initial jet velocity at the nozzle exit.

The right-hand side of the momentum equation is a function of x only and, with $m=1$, the left-hand side is a function of η only. Therefore, each side must be equal to the same constant. The solutions for F and ξ can, thus, be obtained from the two independent equations that comprise the two sides of the momentum equation.

The left-hand side yields

$$F' F' + F F'' + (2d/L_I) F''' = 0$$

when the constant is absorbed into L_I . With $L_I = 4d$, the form of the equation is the same as for the laminar jet. The solution $F = \tanh \eta$ is therefore the same, and a self-preserving velocity distribution is possible. It is not surprising that the shape of the profile is the same, in view of the assumption that there is an effective viscosity for the hypermixing stress. However, the existence of a self-preserving solution could not be predicted, due to the complex coupling between the jet and vortex motions.

The momentum integral will also be used to evaluate the initial velocity constant U_I . With F known, the integration can be performed, yielding $I=4/3$, and the momentum conservation equation solved to find $U_I = 0.433 U_0$.

Transformation Equation

The right-hand side of the differential equation yields an equation for the coordinate transformation $\xi(x)$

$$d\xi/dx = \alpha + [A_c/T(\xi/d)]$$

A similar expression results if the foregoing analysis is extended to other free shear flows. This may be of particular importance due to the growing recognition^{4,5} that the largest eddies of many turbulent flows are vortex-like. The general form of this expression is

$$d\xi/dx = \alpha + (A_c/T U_c L_c)$$

in which $\alpha U_c L_c \sim \xi^{m-n}$ is the ordinary eddy viscosity. The large eddies in most naturally occurring shear flows, wakes, and mixing layers especially, have their axes normal to the plane of the flow and, therefore, cannot be strained. They are simply convected by the main stream, so that $T = T_I + x/U_c$. For the case of the plane wake and the round jet, m equals n . Since ϵ is then constant, the equation is linear and can be integrated directly to give

$$\xi(x) = \alpha x + A_c \ln [(x+x_0)/x_0]$$

The complete solution for the velocity profile in the case of the plane wake was obtained in Ref. 3. For other flow geometries, and in particular the hypermixing jet of the present study, the transformation equation is nonlinear and must be numerically integrated.

The age of the vortices as a function of distance from the nozzle can be written

$$T(x) = T_I + U_I^{-1} \int_0^x (\xi/d)^{1/2} dx$$

since

$$U_c = U_I (\xi/d)^{-1/2}$$

The apparent initial age of the vortices T_I can be determined by assuming that the initial size of the vortices is of the same order as L_I . According to the solution for the vortex motion, the initial radius is

$$r_I = (2 \epsilon_I T_I)^{1/2}$$

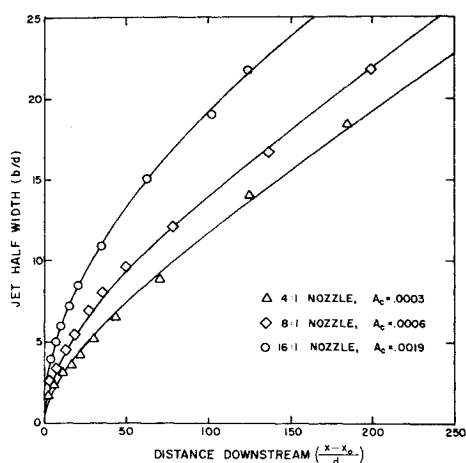


Fig. 3 Comparison of present theory and experiment for hypermixing.

Equating this radius to L_I , with $\epsilon_I = \alpha L_I U_I$, gives for the initial age

$$T_I = 2d/\alpha U_I$$

The initial size and strength of the hypermixing vortices must be specified in order to compute their effect. In practice, of course, the characteristics of the vortices will be inferred from the rate at which the jets are observed to spread or, less elegantly, the free constants α and A_c will be experimentally evaluated.

The characteristic width of the jet will be defined as the width at which $U = \frac{1}{2}U(0)$. Since the self-preserving velocity profile has the form

$$U/U_c = 1 - \tanh^2 \eta$$

the half width L_c is defined by setting $1 - \tanh^2 \eta = \frac{1}{2}$. This yields $L_c(x) = 3.52 \xi(x)$ and, since $d\xi/dx = \alpha$ for large values of x , it is seen that

$$\alpha = (1/3.52)dL_c/dx$$

The constant α can be determined from the jet spreading rate far from the nozzle. In fact, this rate should not be very different from the spreading rate of an ordinary turbulent jet, since the hypermixing vortices must eventually decay. The observed rate is $dL_c/dx = 0.7$ (see Fig. 3), and so $\alpha = 0.02$. The value of A_c was chosen to give the best fit to the data in the region near the nozzle, and indicates the strength of the hypermixing effect.

The solution to the transformation equation was obtained with a conventional Runge-Kutta method of numerical integration. However, since the derivatives of ξ are all infinite at the origin, making it impossible to launch the solution, the inverse equation for x as a function of ξ

$$x'' = A_c^{-1} \xi^{3/2} x' (1 - x' \alpha) + x' \xi^{-1} (1 - x' \alpha)$$

was actually integrated.

The results of this analysis are compared to free-jet data in Fig. 3. All three hypermixing nozzles have the same exit area; they differ in the aspect ratio of the individual segments. The aspect ratio 4:1 nozzle is divided into 10 elements. The 8:1 nozzle has half as many segments, but each is twice as long. The 16:1 nozzle has 2.5 segments. It is believed that the lesser growth rate of the low aspect ratio nozzles is caused by destructive interference between the vortices. In Fig. 4 the isovels of the 8:1 nozzle are shown. The outermost contour corresponds to 50 fps in each case, and the increment between isovels is also 50 fps. The initial segments of the jet can be seen to merge rapidly.

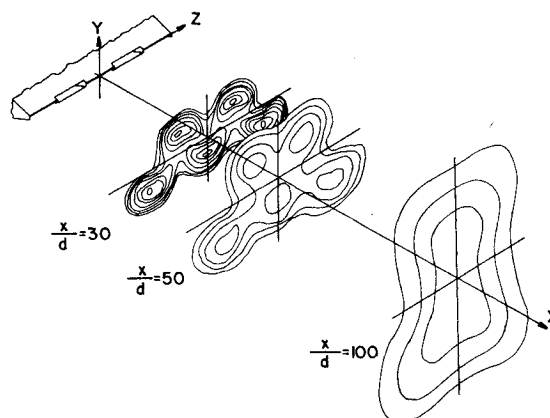


Fig. 4 Isovels of 8:1 hypermixing nozzle. $U_o = 640$ fps.

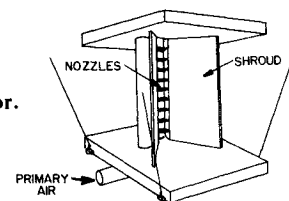


Fig. 5 Sketch of experimental ejector.

It should be understood that the hypermixing eddy viscosity presented is purely descriptive. It cannot predict, for example, the growth of a jet from a 6:1 nozzle; however, it can be used to predict the performance of an ejector which incorporates a nozzle whose growth rate is known.

Experimental Apparatus

The 8:1 hypermixing nozzles were used in the original high inlet area ratio ejector developed at ARL. The exhaust flow was fully mixed within 50 in. and an augmentation ratio in excess of 2 was obtained⁶ with an inlet area ratio of approximately 24.5. However, reductions in ejector length resulted in significant performance losses. As part of the present study of hypermixing, a mixing improvement program was undertaken to increase the performance of short ejectors.

The experimental ejector is supported by four cables as a kind of swing. A sketch of this apparatus is seen in Fig. 5. The absolute thrust is measured with a fixed load cell which bears against the forward surface of the lower platform. The cell is preloaded so that no motion of the swing actually occurs. Air is supplied to the primary manifolds from below. The rate of primary mass flow is determined with a calibrated orifice plate in the supply line. Primary stagnation conditions are measured with a pressure tap and thermocouple in the manifold.

The accuracy of the system was established by testing over a range of pressure ratios with a pair of axisymmetric calibration nozzles whose area was equal to the total of all the hypermixing nozzles. A linear regression analysis of these data points showed the 90% confidence band to be within $\pm 2\%$ of the mean. Other details relevant to the test ejector and instrumentation have been reported in Ref. 6.

Results and Discussion

The strength of the hypermixing effect depends upon the aspect ratio and deflection of the alternating jet segments. The spreading of the jet can be increased by changing either parameter, and both methods were explored in the present study. A hypermixing nozzle with a spreading rate twice that of the early design was fabricated by cutting the short side of the 8:1 nozzles back so that the normal to the exit plane makes

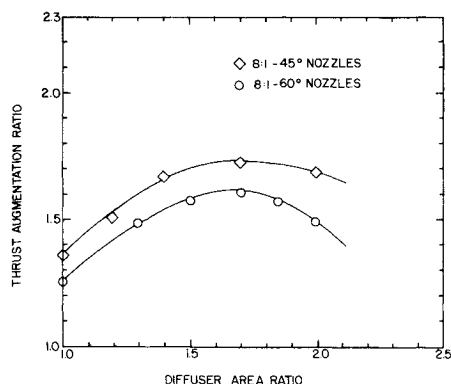


Fig. 6 Measured augmentation showing effect of nozzle exit angle.

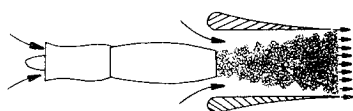


Fig. 7 Simple ejector showing ideal length.

an angle of 60° with the jet centerline. The original design makes a 45° angle.

The thrust efficiency was measured to be $\eta = 0.95$ for the 45° nozzles, and $\eta = 0.91$ for the 60° nozzle. These nozzles were compared in an ejector having a 10-in. mixing section and a 30-in. diffuser. The measured levels of augmentation are shown in Fig. 6. The lines drawn through the data are not the result of analysis, but are intended to aid the eye in linking associated points. It can be seen that the increased mixing of the 60° nozzle does not compensate for its lower thrust efficiency; the augmentation is greater with the original 45° design. This is surprising because the jet from the original nozzle had twice the spreading rate of an ordinary turbulent jet with a 4% loss in thrust efficiency, and the 60° nozzle doubled that rate (giving four times the entrainment of a slot nozzle) with another 4% loss in efficiency. It is apparent that the balance between increased mixing and the loss of efficiency is precarious.

A more successful solution to the problem of enhanced mixing was obtained by increasing the aspect ratio of the nozzle segments. This has the effect of strengthening individual vortices, although the total number of vortices in a given span is reduced. Clearly, there is a balance here, too. The optimum has not yet been identified and in fact, it may vary with the ejector geometry. The jet from the 16:1 hypermixing nozzle previously described was also found to have nearly four times the entrainment of the reference jet with essentially the same efficiency, $\eta = 0.95$, as the 8:1 nozzle.

For a short ejector of fixed length, increased mixing results in higher levels of augmentation. This has been shown in Ref. 7. The amount of augmentation is directly dependent upon the uniformity of the exhaust flow, that is, on the flatness of the velocity profile at the diffuser exit. If the exhaust flow is uniformly mixed, further increases in the rate of entrainment by the primary jet will not increase the augmentation, and can actually decrease it. This may seem strange, but consider the elementary ejector in Fig. 7. The length of the shroud coincides exactly with the point at which the flow becomes fully mixed. Increased mixing does not draw more secondary air through the inlet, but rather moves this point upstream. Larger wall friction forces may then decrease the net thrust. However, it has become possible to decrease the length of the ejector with no loss in performance.

Both the 8:1 and the 16:1 hypermixing nozzles were tested in the 40-in. ejector previously described. The measured augmentation is shown in Fig. 8. There is essentially no difference, because with a duct this long, each nozzle produces a fully mixed flow at the exit. The overall increase in the augmentation above that shown in Fig. 6 is due to the use of

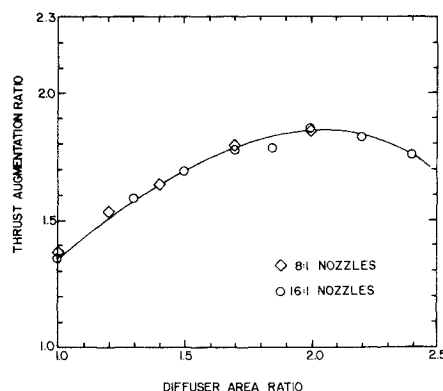


Fig. 8 Comparison of hypermixing nozzles in 40 in. ejector.

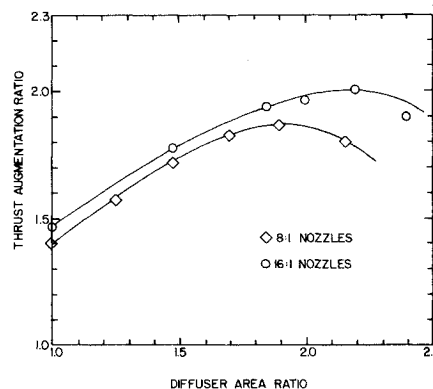


Fig. 9 Comparison of hypermixing nozzles in 30 in. ejector.

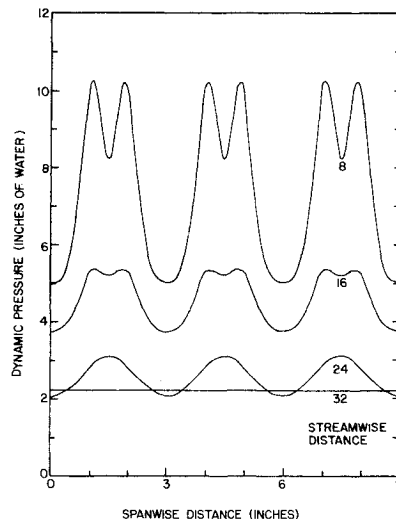


Fig. 10 Dynamic pressure profiles in the mixing-diffuser sections of 30 in. ejector.

better boundary-layer control in the endwalls. Simply, an array of small nozzles was installed in place of the single large nozzle used in the previous configuration.

The performance of the hypermixing nozzles were also compared in an even shorter ejector, having a 5-in. mixing section and a 25-in. diffuser. The shorter mixing section permits more gradual divergence of the diffuser walls for a given exit area, and the flow, therefore, remains attached to a larger diffuser area ratio. The array of endwall nozzles was used. Figure 9 shows the level of augmentation obtained. The flow is not fully mixed with the 8:1 nozzles, and so the diffuser stalls at an area ratio of approximately 1.9. Interestingly, the maximum augmentation is comparable to that measured in the 40-in. ejector. The desirability of a short mixing duct and the resulting gradual divergence of the diffuser walls is clear.

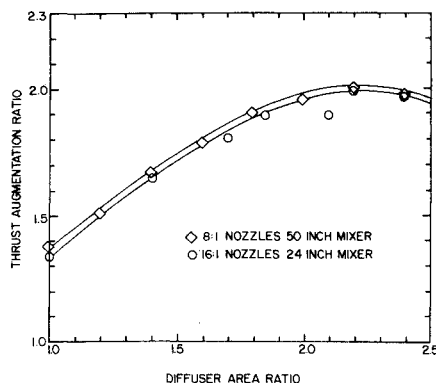


Fig. 11 Comparison of hypermixing nozzles in ejectors of different length.

In Fig. 10, the averaged dynamic pressure profiles produced by the 16:1 nozzles in the same ejector are shown. The profiles were obtained at stations 8, 16, 24, and 32 in. downstream of the nozzle exit plane, and spanned the vertical distance across three adjacent jet wakes. The double peaks in the profiles nearest the nozzle correspond to the divergence of the jet segments seen in Fig. 4. It is apparent that the flow is well mixed by the end of this 30-in. ejector. The augmentation produced by these nozzles is also shown in Fig. 9. The peak value of the augmentation is actually higher than for the 40-in. ejector, illustrating the point about the effect of increased wall friction.

There is an optimum ejector length for a given set of nozzles. If the ejector is made shorter, incomplete mixing reduces the level of augmentation; if the ejector is made longer, the augmentation is reduced by wall friction losses. With the original 8:1 nozzles and a 5-in. mixing section, the optimum diffuser length was approximately 45-in. This has been reduced to 25-in. with the 16:1 nozzles. Since it is desirable to make the ejector as short as possible, a configuration with a 19-in. diffuser section was also tested. In Fig. 11 the performance of this short ejector with the 16:1 nozzles is compared to the best performance obtained with the 8:1 nozzles.

The length of the mixing diffuser section has been reduced by more than 50%, with only a 2% loss in maximum thrust.

There is a final observation to be made about the performance of ejectors in general. The single low point seen in Fig. 11 at the diffuser area ratio of 2.1 was produced by a roughness element about 0.25 in. wide and 0.50 in. high, placed on the side wall just downstream of the injection plane. The flow separated from the diffuser wall over a wide area behind this obstacle, and the augmentation dropped as shown. In building an ejector, care must be taken to insure that surfaces are smooth, and the flow remains attached to the walls. Ejectors are similar to laminar-flow wings in this respect.

Conclusions

The development of an improved hypermixing nozzle has made it practical to halve the length of the thrust augmenting ejector developed at ARL with practically no loss in augmentation. An analytic description of the hypermixing effect can be obtained with an eddy viscosity whose length and velocity scales are proportional to the vortex size and rotational speed.

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